**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B (0.26576)

1-stats.norm.cdf(50,45,8)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: 1-stats.norm.cdf(44,38,6)>( (stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6))

0.15 > 0.34 which is false.

P(X>44)>P(38<X<44)=false

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

stats.norm.cdf(30,38,6)=Probability of Employees who are under the age of 30.

P(X<30)=0.09

E(X<30)=P(X<30)\*400=0.09\*400=36

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.
2. As we know that if X ∼N(μ1, σ1^2 ), and Y ∼N(μ2, σ2^2 ) are two independent random variables then X + Y ∼N(μ1 + μ2, σ1^2 + σ2^2 ) , and X − Y ∼N(μ1 − μ2, σ1^2 + σ2^2 ) .Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).Therefore in the problem 2X1 -X1+X2 which is the linear combination of two variables also gives normal distribution and also the parameters can be 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ^2 + 2σ^2 )
3. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
4. 90.5, 105.9
5. 80.2, 119.8
6. 22, 78
7. 48.5, 151.5
8. 90.1, 109.9

Ans: stats.norm.interval(0.99,100,20)

D 48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Mean Profit=5+7/2=6$\*45=Rs.270 Millions

Standard Deviation=sqrt(16+9)/2=5\*45=Rs.225 Millions

Range of Probability >0.95=stats.norm.interval(0.95,270,225)

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

*To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645*

X**=** 540**+**(**-**1.645)**\***(225)

X=Rs.170 Millions

1. Which of the two divisions has a larger probability of making a loss in a given year?

*Probability of Division 1 making a loss P(X<0):* stats**.**norm**.**cdf(0,5,3)

0.0477903522728147

*Probability of Division 2 making a loss P(X<0)*

stats**.**norm**.**cdf(0,7,4)

0.040059156863817086

So, the probability of division 1 making a loss is more.